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MODELS THAT REFLECT THE VALUE OF
INFORMATION IN A COMMAND AND CONTROL CONTEXT

by

D. P. Gaver
Naval Postgraduate School

October 1980

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The influence of information upon combat models is given an initial investigation. Coordination of unit fire is evaluated on a static, and then dyamic (Lanchesterian) basis. A model is made for a discriminating missile directed at valuable targets surrounded by false (low value) targets.

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MODELS THAT REFLECT THE VALUE OF INFORMATION IN A COMMAND AND CONTROL CONTEXT

D. P. Gaver

A

1. Introduction

The importance of information in military decision making is widely recognized, and the existence of many sophisticated intelligence-gathering and processing systems is a consequence of this recognition. However, there now seem to be few analytical studies that attempt to explicitly relate information to ultimate military success. In this report an attempt is made to investigate some simple conflict situations, the outcomes of which are likely to be strongly influenced by the information possessed by the opponents. The situations selected are simple enough to be analyzed mathematically, at least in a preliminary way, but no attempt is made to thoroughly explore all of their ramifications, especially in mathematical directions. All of the formulations suggested and explored are quite tentative and incomplete; interesting refinements and realistic modifications will suggest themselves to some readers.

One use for models of the type discussed here is to enhance the efficiency, realism, or validity of complex conflict simulations and wargames. At present, combat models such as Lanchester's equations are used in a modular fashion in some wargames to decide isolated confrontation outcomes. The Air Force TAC WARRIOR provides an example. Somewhat different models, such as the ones described here, can serve such a purpose. Another use is to

facilitate quick and simple exploration of tradeoffs between asset types. Questions of the following types may be tentatively addressed: is increased investment likely to be more profitably spent on weapons or on command, control, and communications (C^3) systems? Likewise, tactical options may be investigated: should attacks be directed at weaponry or at command centers? Answers to such questions, even based on oversimplifications, should be of help in suggesting improved defensive and offensive tactics. If necessary, deeper probing by more elaborate wargaming can be employed for backup confirmation. The kinds of models proposed here are tentative and suggestive, and have no definitive pretensions.

2. Combat With and Without Coordination

2.1. The Problem Area

The purpose of this section is to suggest some extremely simple models of combat that represent the effects of coordination or information sharing. Our approach will first be to study the influence of lack of coordination upon the attrition power of one force against another, and then to compare this with the increased attrition power obtained under coordination--the latter being made possible by the information flow characteristic of a C^3 system.

2.2. Model 1: Static Salvo Interchange Formulation

Suppose a group of B forces confront one of R (we use the symbols B and R variously to label the forces, or to refer to a generic member of the respective forces, or to enumerate the initial force sizes; the context tells the tale). Suppose B wishes to attack R, and does so without coordination, i.e. each B picks a member of R at random and fires at it once, independently of the behavior of the other B's. For the moment assume that all R's are equally likely to receive fire from a single B. Also assume that the kill probability of B against R is unity; this assumption is extreme but convenient and somewhat informative, and will later be relaxed.

Obviously the lack of coordination or information transfer among B's leads to inefficiency: some R's will receive two or more of B's shots and experience overkill; others may receive none, surviving by chance neglect. As a measure of the effectiveness of such fire on the part of B, calculate

2.3. The Expected Number of R's Hit

This is a classical "occupancy problem" (see Feller [1966]) that can be solved by use of indicator functions. If X_R is the random variable denoting the number of Rs hit by B missiles after one B salvo, note that

$$X_R = \ell_1 + \ell_2 + \ldots + \ell_R \tag{2.1}$$

where the indicator

$$\ell_j = \begin{cases} 1 & \text{if } j\underline{th} \ R \text{ is hit by } B \text{ fire} \\ 0 & \text{otherwise.} \end{cases}$$

Now by the linearity of the expectation operator,

$$E[X_R] = \sum_{j=1}^{R} E[\ell_j] \ . \tag{2.2}$$

Since each indicator has the same marginal distribution, we need only calculate the probability of at least one hit on j (recall that kill probability is temporarily one): the probability that all B shots are directed elsewhere is $[(R-1)/R]^B$ and so

$$P\{\ell_j=0\} = \left(1 - \frac{1}{R}\right)^B , \tag{2.3}$$

while

$$P\{\ell_j=1\} = 1 - P\{\ell_j=0\} = 1 - \left(1 - \frac{1}{R}\right)^B ,$$

and, therefore, it follows that the expected number of R's hit under uncoordinated attack is

$$E[X_R] = R\left[1 - \left(1 - \frac{1}{R}\right)^B\right] . \tag{2.4}$$

Calculation of the variance, and the entire distribution is also manageable and only a little more complicated. Under certain conditions the distribution of the latter X_R, properly normalized will approach the normal or Gaussian form (see Sevastyanov and Christyakov [1978]).

It is also possible to derive a formula for the situation in which the probability of a B hitting each R depends upon <u>which</u> R is fired upon. That is, suppose each B picks the j<u>th</u> R with probability r_j. Then the probability that no B picks the j<u>th</u> R is $(1-r_j)^B$, and, following the earlier pattern,

$$E[X_R] = \sum_{j=1}^{R} [1-(1-r_j)^B] . \tag{2.5}$$

It is even possible to calculate the expected number of R's hit if the probability that the i<u>th</u> B picks the j<u>th</u> R independently is r_{ij}; note that such differences may be caused by different intervisibilities, possibly reflecting terrain effects. In this case the probability that the j<u>th</u> R is <u>not</u> picked is

$$(1-r_{1j})(1-r_{2j})\ldots(1-r_{Bj}) = \prod_{i=1}^{B} (1-r_{ij})$$

and, adding up over the j R's, we find that

$$E[X_R] = \sum_{j=1}^{R} [1-\prod_{i=1}^{B} (1-r_{ij})] \tag{2.6}$$

For the moment we stick with the simple model (2.4) for discussion.

It is instructive to look at the ratio

$$\frac{E[X_R]}{R} = \text{Expected fraction of } R\text{'s hit}$$

as the latter depends upon the initial ratio of B to R: $B/R = \beta$. From (2.4),

$$\frac{E[X_R]}{R} = \left[1 - \left(1 - \frac{1}{R}\right)^{\beta R}\right] \to 1 - e^{-\beta} \tag{2.7}$$

if B (and R) become large. This is very simple and handy, and leads to an immediate assessment of the effect of lack of coordination or information transfer, for by our assumptions if B ($= \beta R$ in number) fires in a coordinated fashion at R, i.e. each B has only one R target, then $E[X_R] = \beta R$, provided $\beta \le 1 (B \le R)$, while $E[X_R] = R$ if $\beta \ge 1 (B > R)$. If we assess the advantage of coordination by $A(\beta)$, then in the light of the previous comments,

$$A(\beta) = \frac{E[X_R \mid \text{With Coordination}]}{E[X_R \mid \text{Without Coordination}]} = \begin{cases} \dfrac{\beta}{1-e^{-\beta}}, & 0 \le \beta \le 1 \\ \dfrac{1}{1-e^{-\beta}}, & \beta > 1 \end{cases} \tag{2.8}$$

Here is a sketchy numerical table to illustrate the gain from coordination at constant B-to-R ratio (β) when B and R are large.

β	$A(\beta)$
0.2	1.10
0.4	1.21
0.6	1.33
0.8	1.45
1.0	1.58
1.2	1.43
1.4	1.33
1.6	1.25

Table 1.

A graph appears below

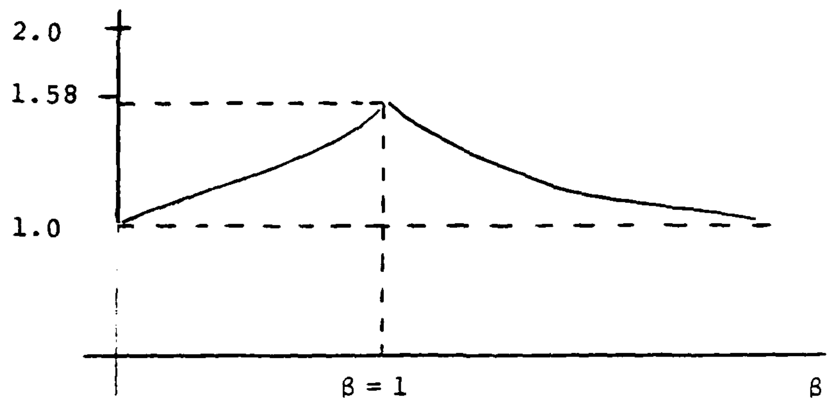

Figure 1.

In this first simple model coordination pays off most when the forces are about equally numerous: if B is much smaller than R then the chances of random overlap are small, and so coordination is not required, while if B greatly outnumbers R coordination will again not be required to assure coverage. Under the latter conditions there is extensive overkill, and some B forces can be usefully employed elsewhere.

2.4. The Expected Number of B's Hit

The above model merely calculates the effect of a B action against R. If we assume that R fires simultaneously at B then the corresponding expected number of B's hit is, from (2.4) by symmetry.

$$E[X_B] = B\left[1 - \left(1 - \frac{1}{B}\right)^R\right] . \tag{2.9}$$

The kill probability is still assumed to be unity in this model.

2.5. The Expected Number of R's Killed in a Single Engagement, Both Without, and With, Coordination

Here is a generalization of the earlier models to explicitly account for (i) the less-than-unit kill probability of R by B, denoted by p_{RB}, and (ii) the rate of fire of B, denoted by ρ_B; similar calculations can be made for B and R.

Let X_R denote the number of R's killed by B's per engagement, during which period $\rho_B B$ shots by B take place. Again use the representation

$$K_R = \ell_1 + \ell_2 + \ldots + \ell_j + \ldots \ \ell_R \qquad (2.10$$

where

$$\ell_j = \begin{cases} 1 & \text{if } \underline{jth} \ R \text{ is } \underline{\text{killed}} \text{ by } B \text{ fire} \\ 0 & \text{otherwise .} \end{cases} \qquad (2.11)$$

Since

$$E[K_R] = \sum_{j=1}^{R} E[\ell_j] = RE[\ell_j] = R \cdot P\{\ell_j = 1\} \qquad (2.12)$$

by the assumptions (i) and (ii), it is only necessary to compute the expectation $E[\ell_j]$.

Follow these steps to compute the expectation:

(A) The probability that i of the B's target on a particular (the $\underline{jth}$) R is given by the binomial

$$\binom{B}{i}\left(\frac{1}{R}\right)^{i}\left(1-\frac{1}{R}\right)^{B-i}, \qquad i = 0,1,\ldots,B \qquad (2.13)$$

(B) Given that a B is targetted on the jth R, the probability of at least one killing event (a kill) in a time interval of length δ (engagement duration) is

$$1 - (1-p_{RB})^{\rho_B \delta} .$$

Consequently the probability of at least one killing event (kill) by i B's firing independently is

$$1 - \left[(1-p_{RB})^{\rho_B \delta}\right]^i = 1 - z^i , \qquad (2.14)$$

where z may be interpreted as the probability of no kill (survival) per engagement with one B.

(C) Now remove the condition on the number of B's:

$$P\{\ell_j = 1\} = \sum_{i=0}^{B} (1-z^i) \binom{B}{i} \left(\frac{1}{R}\right)^i \left(1 - \frac{1}{R}\right)^{B-i}$$

$$= 1 - \left(z \cdot \frac{1}{R} + 1 - \frac{1}{R}\right)^B \qquad (2.15)$$

or

$$E[\ell_j] = P\{\ell_j = 1\} = 1 - \left\{1 - \frac{1}{R}\left[1 - (1-p_{RB})^{\rho_B \delta}\right]\right\}^B .$$

It follows from (2.12) that

$$E[K_R] = R\left[1 - \left\{1 - \frac{1}{R}\left[1 - (1-p_{RB})^{\rho_B \delta}\right]\right\}^B\right] , \qquad (2.16)$$

the expected number of R's killed by B's in an engagement of duration δ. This expression generalizes (2.4) to account for less than unit kill probability and also allows the fire rate ρ_B to be specified. It continues to assume that all R's are

equally susceptible to B targetting, i.e. that there is no coordination. Note that this does not necessarily represent minimum coordination, for it might well be that the R's could arrange for most or all B's to target only members of a very small subset of the R's--perhaps made up of valueless decoys. The tools for evaluating such a capability are at hand. See Section 3 of this report.

Next assess the advantage of coordination by B when opposing R. It is worth remarking that coordination here means that B shots are shared as equally as possible across R's. This tactic seems sensible as long as p_{BR} is high and a reasonable number of shots can be gotten off. Otherwise, a deliberate "gang up" tactic might be worth-while, and could be evaluated.

(a) $\frac{B}{R} = \beta$ (constant); $B, R \to \infty$.

Without coordination we have for fixed δ the expression (2.16); with coordination we have

$$E[K_R] = B\left[1 - (1-p_{RB})^{\rho_B \delta}\right] \quad \text{for } \frac{B}{R} = \beta \le 1$$
$$= R\left[1 - (1-p_{RB})^{\rho_B \beta \delta}\right] \quad \text{for } \frac{B}{R} = \beta > 1 \; ; \tag{2.17}$$

the latter, second, formula assumes that $(B/R = \beta)$ B's are allocated to each R during δ. This may be called an even distribution coordination tactic. The latter, first, formula assumes allocation of one B per R as long as they last. Some R's are left unattended.

It follows that when both B and $R \to \infty$ (both forces are large, but in constant ratio)

$$A(\beta) = \frac{E[K_R|\text{Coordination}]}{E[K_R|\text{Without Coordination}]}$$

$$= \frac{\beta\{1-(1-p_{RB})^{\rho_B\delta}\}}{1-\exp\{-\beta[1-(1-p_{RB})^{\rho_B\delta}]\}}, \quad 0 \le \beta \le 1$$

$$= \frac{\{1-(1-p_{RB})^{\rho_B\delta\beta}\}}{1-\exp\{-\beta[1-(1-p_{RB})^{\rho_B\delta}]\}}, \quad \beta > 1 \tag{2.18}$$

It is important to note that we are assuming no opportunity to check for the effect of a shot during time δ and change aim point if successful, so no advantage is shown for rapid fire rate, high kill probability and re-direction. It is assumed that at the beginning of the engagement interval δ each available target is acquired.

It is tempting to compare the above models for attrition with and without coordination when the engagement length, δ, becomes small. Unfortunately the present models give indistinguishable results in this limit.

(b) B, R fixed, engagement time $\delta \to 0$.

To study attrition of R's without B coordination rewrite (2.16) and expand in power series:

$$E[K_R] = R\left[1-\left\{1-\frac{1}{R}\left[1-e^{\ln(1-p_{RB})\rho_B\delta}\right]\right\}^B\right]$$

$$= R\left[1-\left\{1+\frac{1}{R}\left[\ln(1-p_{RB})\rho_B\delta+o(\delta)\right]\right\}^B\right]$$

$$= R\left[1-\left\{1+\frac{B}{R}\ln(1-p_{RB})\rho_B\delta\right\}\right]+o(\delta)$$

$$= -B\ln(1-p_{RB})\rho_B\delta+o(\delta)\ . \tag{2.19}$$

With coordination we may use (2.17); expanding in power series we get precisely the result (2.19). Another, halfway, approach to coordination would be to split the R's into k disjoint subgroups of size R/k each, and then assign B/k of the B's to each group. Let $K_{R/k}$ denote the attrition in the generic subgroup. Then total expected attrition is

$$E[kK_{R/k}] = (k) \cdot \frac{R}{k}\left[1 - \left\{1 - \frac{1}{R/k}\left[1 - (1-p_{RB})^{\rho_B\delta}\right]\right\}^{B/k}\right] , \qquad (2.20$$

and again a power series expansion shows that as $\delta \to 0$,

$$E[kK_{R/k}] = -B\,\ell n(1-p_{RB})\rho_B\delta + o(\delta) . \qquad (2.21)$$

The conclusion is that in the limit the present model does not reflect the advantage of coordination over a short time interval.

Note that the present simple model contains no explicit agent for gathering and disseminating command and control information. Some recognition of the cost of C^3 can be introduced by depleting the firing rate (in this case of R) of the coordinated side to account for time spent in C^3 activity. But more explicit models are apt to be more informative; some are under development.

2.6. The Number of R Killed in a Single Engagement: A Recursion Approach

It may be of interest to describe an alternative approach to representing the attrition of R's by B's (and vice versa). Think of B's being assigned sequentially to the R's so that $K_R(j)$ is the number of R's killed after exactly j B's have fired their salvos in an uncoordinated manner. Then notice that

A simple recursion (Markov chain) describes the situation:

$$K_R(j) = K_R(j-1) + \begin{cases} 1 & \text{with probability } \left(1 - \frac{K_R(j-1)}{R}\right)\bar{z} \\ 0 & \text{with probability } 1 - \left(1 - \frac{K_R(j-1)}{R}\right)\bar{z} \end{cases} \tag{2.22}$$

The argument is that the number of killed R's increases by one on the addition of the j<u>th</u> B salvo if and only if (i) the j<u>th</u> B targets an R not yet killed (probability: $1-\frac{K_R(j-1)}{R}$ according to equal likelihood) and (ii) the j<u>th</u> B's salvo results in at least one killing hit (probability $\bar{z} = 1-(1-p_{RB})^{\rho_B\delta}$). Notice that (2.22) shows that $\{K_R(j)\}$ is a Markov chain, and that the representation can be used to easily <u>simulate</u> R attrition for any number of B's. Of course the same procedure can be used to generate or simulate B attrition.

Take expectations to re-derive (2.16):

$$E[K_R(j)|K_R(j-1)] = K_R(j-1) + \left(1 - \frac{K_R(j-1)}{R}\right)\bar{z}$$

$$= \bar{z} + K_R(j-1)\left(1 - \frac{\bar{z}}{R}\right), \tag{2.23}$$

and

$$E[K_R(j)] = \bar{z} + \left(1 - \frac{\bar{z}}{R}\right)E[K_R(j-1)]$$

Start with $j = 2$; $E[K_R(1)] = \bar{z}$, and recurrence provides

$$E[K_R(j)] = R\left[1 - \left\{1 - \frac{\bar{z}}{R}\right\}^B\right] \tag{2.24}$$

which is precisely (2.16).

The representation (2.22) allows the derivation of a recursion for the variance of $K_R(j)$. Begin by squaring (2.2):

$$K_R^2(j) = K_R^2(j-1) + \begin{cases} 2K_R(j-1) + 1 & \text{with probability } \left(1 - \frac{K_R(j-1)}{R}\right)\bar{z} \\ 0 & \text{with probability } 1-\left(1 - \frac{K_R(j-1)}{R}\right)\bar{z} \end{cases} \quad (2.25)$$

Again take expectations:

$$\begin{aligned} E[K_R^2(j) \mid K_R(j-1)] &= K_R^2(j-1) + (2K_R(j-1)+1)\left(1 - \frac{K_R(j-1)}{R}\right)\bar{z} \\ &= \bar{z} + \left(1 - \frac{2\bar{z}}{R}\right) \cdot K_R^2(j-1) + \bar{z}\left(2 - \frac{1}{R}\right) K_R(j-1) \ . \end{aligned} \quad (2.26)$$

Now remove the conditions to find

$$E[K_R^2(j)] = \bar{z} + \left(1 - \frac{2\bar{z}}{R}\right) E[K_R^2(j-1)] + \bar{z}\left(2 - \frac{1}{R}\right) E[K_R(j-1)] \ . \quad (2.27)$$

Of course the variance is simply

$$\mathrm{Var}[K_R^2(j)] = E[K_R^2(j)] - (E[K_R(j)])^2 \ . \quad (2.28)$$

One can compute $E[K_R^2(j)]$ recursively from (2.27), knowing the formula (2.24) or using (2.23). A closed-form expression can be obtained if desired, but we skip this exercise for the result will be very complex. Very likely $K_R(B)$, suitably normalized is normally distributed (Gaussian) for large R and B. See Gaver and Powell (1971) for a similar application of occupancy models that invokes alternative approaches.

2.7. A Dynamic Attrition Model for Assessing the Value of Coordination

It is tempting to write down Lanchester-type attrition models to represent the course of combat between two opposed

forces when one side coordinates and the other does not. The expressions for $E[K_R]$ (and $E[K_B]$ already derived can be used for this, at least in a rough preliminary way.

The attrition equations will be developed in discrete time, with units of time advance taken to be $\delta = 1$. Imagine that initially B's force size is B, and R's is R, and let $R(t)$ and $B(t)$ represent force sizes after t engagements or interchanges. Assume that B is uncoordinated in its engagement with R, but R is coordinated against B. Our present model treats combat as a sequence of individual engagements with deterministic outcomes. Here are the equations:

$$\begin{aligned} R(t+1) &= R(t) - R(t)\left[1 - \left\{1 - \frac{1}{R(t)}\left[1 - (1-p_{RB})^{\rho_B}\right]\right\}^{B(t)}\right] \\ &= R(t)\left\{1 - \frac{1}{R(t)}\left[1 - (1-p_{RB})^{\rho_B}\right]\right\}^{B(t)} ; \end{aligned} \tag{2.29}$$

next,

$$\begin{aligned} B(t+1) &= B(t) - R(t)\left[1 - (1-p_{BR})^{\rho_R}\right] \\ &= B(t) - R(t)\left[1 - (1-p_{BR})^{\rho_R}\right] \quad \text{if} \quad R(t) \le B(t) . \end{aligned} \tag{2.30}$$

However,

$$\begin{aligned} B(t+1) &= B(t) - B(t)\left[1 - (1-p_{BR})^{\rho_B R(t)}\right] \\ &= B(t)(1-p_{BR})^{\rho_B R(t)} \quad \text{if} \quad R(t) > B(t) . \end{aligned} \tag{2.31}$$

These equations, together with the initial conditions $R(0) = R$, $B(0) = B$ can be used to generate the entire deterministic history

of a combat. Suitably modified, they can generate a stochastic realization or simulation. Numerical illustrations follow. Notice that there would be no difficulty with making p_{BR}, p_{RB} or even ρ_R and ρ_B depend upon the time, t.

2.8. Numerical Examples of Dynamic Attrition

The simple discrete-time Lanchesterian equations (2.29), (2.30), and (2.31) are exceptionally easy to solve on a computer. A program has been written to do so, and is available on the NPS system via interactive terminal. We report a few isolated experiments to show the effects of coordination.

Begin with

Case I: $p_{RB} = p_{BR} = 0.5, \quad B(0) = R(0) = 20$

$$\rho_R = \rho_B = 1 .$$

Here the two sides are evenly matched from a physical viewpoint, but B is Uncoordinated in fire against B, while R is Coordinated. Here is the result

t	R(t)	B(t)
0	20	20
1	12	10
2	8	0*

Table 2

where rounding* is to the nearest integer. Clearly the battle goes decisively to the Coordinated combatant, and quickly so:

Case II: Again $p_{RB} = p_{BR} = 0.5$

$$\rho_R = \rho_B = 1$$

We summarize for different initial forces for B:

t	R(t)	B(t)	R(t)	B(t)	R(t)	B(t)	R(t)	B(t)
0	20	20	20	22	20	23	20	25
1	12	10	11	12	11	13	11	15
2	8	0*	7	1	6	7+	5	10
3	- - -	- - -	4	0*	3	4	2	7
4	- - -	- - -	- - -	- - -	2	3	0*	6
5	- - -	- - -	- - -	- - -	0*	2		

Table 3

Note that in the present hypothetical situation B needs an advantage of 3 to win, and the battle is much prolonged. The situation is probably unrealistic in that the model suggests very heavy attrition in the first interchange. But this effect must be largely attributable to the assumed very high kill probability levels. Here, by way of contrast, is

Case III: $p_{RB} = p_{BR} = 0.2$

$$\rho_R = \rho_B = 1$$

Summarize for different initial forces for B:

t	R(t)	B(t)	R(t)	B(t)	R(t)	B(t)
0	20	20	20	21	20	22
1	16	14	16	17	16	18
2	14	0*	13	14	13	15
3			11	11	10	12
4			9	9	8	10
5			7	7	6	9
6			6	6	5	7
7			5	5	3	7
8			4	4	2	6
9			3	3	1	5
10			2	3	0*	5
11			2	2		
12			2	2		
13			1	1		
14			1	1		

Table 4.

Note that in this case of lower kill probability B needs only advantage of 2 to win, so that coordination has smaller leverage.

Of course the above analysis is entirely deterministic, and a careful stochastic-model analysis might well add further insights. As it turns out, the recursion relations (2.29)-(2.31) can be converted to stochastic difference equations that can form the basis for a stochastic simulation of mutual force attrition. This idea will be investigated in future work.

2.9. Another Model Involving C^3 Activity: Assignment with Checking

We present another model that represents the problem of targetting a finite force, B, upon another, R. The model and

methodology are slightly different from the previous approaches. This time we conduct the calculations recursively, imagining the B's to be allocated one at a time to the R's.

Let $X_R(k)$ be the number of distinct R's targetted by B's after k of the B's have conducted targetting activities; $k = 1,2,\ldots,B$. Here targetting consists of (a) initial target selection (at random), and then (b) checking, and correction. The idea of the second step is to avoid duplicate coverage (if technically feasible). The checking step is conducted with error, and may be viewed as a C^3 function: the more effective the C^3, the smaller will be the undesired duplicate coverage.

The first phase of targetting is initial target selection, which is assumed to be done at random, in the sense that each R unit has an equal chance of being present after this phase (the k<u>th</u>), whether or not it was before, that is regardless of whether it has been targetted after $k-1$ B's are assigned.

The second phase may be termed <u>checking</u>, and has a command-control flavor: if at the targetting time it is possible to detect the possibility that another B has already targetted the particular R selected, then a switch is made to a previously untargetted R. Let this sequence take place with probability θ. Then $\theta = 1$ represents full C^3 coordination capacity, and $\theta = 0$ represents total absence of such capacity; $\bar{\theta} = 1 - \theta$ represents the probability of failure of checking. The present model assumes that <u>if</u> duplication is discovered it can be avoided with certainty. A second model avoids this assumption but turns out to be rather unmanageable analytically, although a simulation approach suggests itself.

(A) Random targetting with one recourse step.

The following recursive expression describes $X_R(k)$:

$$X_R(k) = X_R(k-1) + \begin{cases} 0 & \text{with probability } \dfrac{X_R(k-1)}{R}\,\bar{\theta} \\ 1 & \text{with probability } 1 - \dfrac{X_R(k-1)}{R}\,\bar{\theta} \end{cases} \qquad (k=1,2,3,\ldots,B) \tag{2.32}$$

so long as $X_R(k) \le R$ (obviously since $X_R(0) = 0$, $X_R(k) \le k$). The idea is that if there are $X_R(k-1)$ B's allocated to R's after $k-1$ targetting steps then the next step is unsuccessful in adding a <u>new</u> R if a) the <u>kth</u> random selection is of a designated target (probability $= \dfrac{X_R(k-1)}{R}$), and b) the checking procedure is unsuccessful (probability $= \bar{\theta}$). Assumption of independence leads to the first line of (2.32), and the complementary probability gives the second line. Now take conditional expectations to find

$$E[X_R(k)\,|\,X_R(k-1)] = X_R(k-1) + 1\cdot\left[1 - \frac{X_R(k-1)}{R}\,\bar{\theta}\right] . \tag{2.33}$$

and hence

$$E[X_R(k)] = E[X_R(k-1)] + 1 - \frac{\bar{\theta}}{R}\,E[X_R(k-1)]$$

$$= 1 + \left(1 - \frac{\bar{\theta}}{R}\right) E[X_R(k-1)] .$$

Of course,

$$E[X_R(0)] = 0 ,$$

$$E[X_R(1)] = 1 ,$$

and hence by induction in (2.29)

$$E[X_R(2)] = 1 + \left(1 - \frac{\bar{\theta}}{R}\right) \quad ,$$

$$\cdots\cdots \tag{2.35}$$

$$E[X_R(k)] = 1 + \left(1 - \frac{\bar{\theta}}{R}\right) + \left(1 - \frac{\bar{\theta}}{R}\right)^2 + \ldots \left(1 - \frac{\bar{\theta}}{R}\right)^{k-1} = \frac{R}{\bar{\theta}}\left[1 - \left(1 - \frac{\bar{\theta}}{R}\right)^k\right]$$

Thus if B R we have

$$E[X_R(B)] = \frac{R}{\bar{\theta}}\left[1 - \left(1 - \frac{\bar{\theta}}{R}\right)^B\right] \quad , \tag{2.36}$$

and if $\bar{\theta} \to 0$ or $\theta \to 1$ to signify perfect ability to check and switch, then the above expectation approaches B. If $\bar{\theta} = 1$, meaning that there is no coordination capability, we are back to the original formula (2.4). We do not discuss the case $E > R$ for this model. With some added complication the effect of rate of fire and hit probability may be introduced to account for expected hits.

(B) Targetting with finite (geometric) recourse.

Suppose that a redundant targetting is detectable with probability θ, independently from occasion to occasion. We wish to calculate the probability that $X_R(k) - X_R(k-1) = 0$, i.e. the k<u>th</u> allocation is redundant. This happens if there are $n(n=1,2,\ldots)$ random selections, each of which results in a redundant selection, and each of which is finally undetected: the probability that exactly n steps go on is

$$\left(\frac{X_R(k-1)}{R}\theta\right)^{n-1}\left(\frac{X_R(k-1)}{R}\cdot\bar{\theta}\right) \quad ; \tag{2.37}$$

sum over mutually exclusive alternatives to obtain $\frac{X_R(k-1)}{R}\bar{\theta} \Big/ \left\{1 - \frac{X_R}{R}(k-1)\,\theta\right\}$. This then replaces $\frac{X_R(k-1)}{R}\bar{\theta}$ in the recursive expression (2.32). But it seems next to impossible to obtain further analytic information from this model, and so it is hereby dropped from further discussion. Of course the revised recursion may be utilized for simulation if desired.

3. An "Over the Horizon Problem" with a Moderately Intelligent Missile

3.1. The Problem

Suppose one is called upon to shoot a single missile at a far-distant, "over-the-horizon" target. Moreover, there are apt to be other targets of no value within the area of interest. These false targets may well distract the missile, thus rendering it useless. Some calculations will now be made that indicate the chance of hitting (and killing) the target, as the latter depends upon the number of surrounding targets, and--a new feature--the ability of the missile to discriminate between false and true (valuable) targets.

3.2. The Simplest Model

If a fixed number, N, of false targets are near the true target in the area, and if the missile essentially picks one at random (or with equal likelihood) then the probability of correct attack is

$$P_{CA} = \frac{1}{N+1} \tag{3.1}$$

Note that if there are t true targets in the area the probability of a correct attack on one is

$$P_{CA} = \frac{t}{N+t} \tag{3.2}$$

under the same conditions.

3.3. A Moderately Intelligent Missile

Suppose that a missile, or missile plus guidance from satellite or another sensor system can be designed that has the following

discriminatory behavior: a potential target is shown to the missile; if it is (a) false the missile does not attack it with $\bar{\beta}$, but (mistakenly) attacks it with probability β $(\bar{\beta} + \beta = 1)$, while if the target is (b) true or genuine, the missile mistakenly disdains it with probability $\bar{\alpha}$, and correctly attacks it with probability α. All of this is independent of the numbers of times the missile has seen the particular target (the beast has no memory). How has discrimination of this rudimentary kind improved the previous situation?

The missile may be thought of as picking a target at random from the $N + 1$ present, and then deciding whether or not to attack. On a given selection occasion the missile disdains the target with probability

$$\frac{N}{N+1}\,\bar{\beta} + \frac{1}{N+1}\,\bar{\alpha} \tag{3.3}$$

for either a false target occurs with probability $N/(N+1)$ and then is not attacked with probability $\bar{\beta}$ <u>or</u> a true target is picked with probability $1/(N+1)$ and the decision is made not to attack with probability $\bar{\alpha}$. Now at the end of n trials (looks at targets) the missile has still not committed itself with probability

$$\left(\frac{N}{N+1}\,\bar{\beta} + \frac{1}{N+1}\,\bar{\alpha}\right)^{n} \qquad (3.4$$

since the missile is indiscriminate in picking its next candidate. The probability of correct attack is the probability that the missile has remained uncommitted for $n = 0,1,2,\ldots$ looks, but finally on the n+1<u>st</u> picks the correct target and attacks; thus

$$P_{CA} = \sum_{n=0}^{\infty}\left(\frac{N}{N+1}\bar{\beta} + \frac{1}{N+1}\bar{\alpha}\right)^n \frac{1}{N+1}\alpha$$

$$= \frac{1}{1 - \frac{N}{N+1}\bar{\beta} - \frac{1}{N+1}\bar{\alpha}} \frac{1}{N+1}\alpha = \frac{\alpha}{N\beta + \alpha} = \frac{(\frac{\alpha}{\beta})}{N + (\frac{\alpha}{\beta})} \tag{3.5}$$

It seems reasonable to name the ratio α/β the discrimination of the missile (or the system), so (3.5) amounts to

$$P_{CA} = \frac{\text{discrimination}}{N + \text{discrimination}} \tag{3.6}$$

Now note that the discrimination is equivalent to a certain effective number of true targets. For instance let

$$\frac{\alpha}{\beta} = \frac{\text{Prob\{attack true target, given a true target\}}}{\text{Prob\{attack false target, given a false target\}}}$$

have the value t, where t may be in the range $[0,\infty)$, but should be in the range $[1,\infty)$; then

$$P_{CA} = \frac{t}{t+N} \tag{3.7}$$

which is entirely equivalent to (3.2). This equivalence only works for the first shot, if more than one is contemplated. A consideration of the problem of dispatching more than one missile at a group of targets might involve some interesting coordination options. This problem is dodged for the moment.

3.4. Variable Numbers of False Targets

The above problem can be generalized, and possibly made more interesting and informative, by assuming that the number of false targets (e.g. decoys) are variable. In fact, take the plunge and assume that N is a random variable with probability mass function

$$P\{N = n\} = p_n \qquad (n=0,1,2,\ldots) \ . \tag{3.8}$$

Then it is legitimate to calculate the probability of correct attack by considering $P_{CA}(N)$ in the formulas (3.1), (3.2) and (3.5) as being conditional probabilities, given the value of N, and then removing the condition. Thus the generalization needed is to find

$$P_{CA} = \sum_{n=0}^{\infty} \frac{t}{n+t} p_n \ .$$

This calculation is very easily carried out in the case of no discrimination: from (3.1) now

$$P_{CA} = \sum_{n=0}^{\infty} \frac{1}{n+1} p_n \ . \tag{3.9}$$

For example, let N be Poisson,

$$p_n = e^{-\rho} \frac{\rho^n}{n!} \ , \tag{3.10}$$

where ρ is the mean number of false targets in the area. Then

$$P_{CA} = \sum_{n=0}^{\infty} \frac{1}{n+1} e^{-\rho} \frac{\rho^n}{n!} = \sum_{n=0}^{\infty} \frac{1}{(n+1)!} \rho^{n+1} \left(\frac{e^{-\rho}}{\rho}\right)$$

$$= (e^{\rho} - 1) \frac{e^{-\rho}}{\rho} = \frac{1}{\rho} (1 - e^{-\rho}) \ . \tag{3.11}$$

Another mathematical approach to this problem is through the generating function of the distribution of false targets. It

turns out that <u>integrating</u> the generating function is the right move. Let the generating function be

$$p(z) = \sum_{n=0}^{\infty} z^n p_n \qquad 0 \le |z| \le 1 .$$

Then integrate between 0 and 1 to find

$$\int_0^1 p(z)\,dz = \sum_{n=0}^{\infty} \left.\frac{z^{n+1}}{n+1}\right|_0^1 p_n = \sum_{n=0}^{\infty} \frac{1}{n+1} p_n = P_{CA} . \tag{3.12}$$

Try this for the case of N Poisson to establish its credentials: we know that

$$p(z) = e^{-\rho(1-z)} , \tag{3.13}$$

so

$$\int_0^1 e^{-\rho(1-z)}dz = e^{-\rho} \int_0^1 e^{\rho z}dz = \frac{e^{-\rho}}{\rho} (e^{\rho} - 1) = \frac{1-e^{-\rho}}{\rho} \tag{3.14}$$

as before. Other distributions can be handled in the same general way.

Now try the same trick on the mathematically equivalent (3.2). Some modifications are necessary; first scrutinize

$$P_{CA} = \sum_{n=0}^{\infty} \frac{t}{n+t} p_n . \tag{3.15}$$

Note that if we write

$$\sum_{n=0}^{\infty} z^{n+t-1} p_n = z^{t-1} p(z) \tag{3.16}$$

and then integrate

$$\int_0^1 z^{t-1}\, p(z)dz = \sum_{n=0}^{\infty} \left(\int_0^1 z^{n+t-1}\, dz\right)p_n = \sum_{n=0}^{\infty} \frac{1}{n+t}\, p_n \qquad (3.17)$$

and finally multiply by t the desired result follows:

$$P_{CA} = t \int_0^1 z^{t-1}\, p(z)dz \ . \qquad (3.18)$$

With luck this can be evaluated, or be found tabled. Otherwise, it is back to numerical summation, as in (3.15).

Let us try this method on the Poisson distribution of false targets with $t = 2$ (with either two true targets present, or a discrimination of $\alpha/\beta = 2$). Now from (3.18)

$$\begin{aligned} P_{CA} &= 2 \int_0^1 ze^{-\rho(1-z)} = 2e^{-\rho} \int_0^1 e^{\rho z} z\, dz \\ &= 2e^{-\rho} \frac{d}{d\rho} \int_0^1 e^{\rho z}\, dz \\ &= 2e^{-\rho} \frac{d}{d\rho} \left[\frac{e^{\rho} - 1}{\rho}\right] = 2e^{-\rho} \left[\frac{e^{\rho} - (e^{\rho}-1)}{\rho^2}\right] \\ &= \frac{2}{\rho^2} [e^{-\rho} - 1 + \rho] \ . \end{aligned} \qquad (3.19)$$

One can easily do $t = 3,4,\ldots$ <u>in principle</u> although actual results are increasingly messy.

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